

COMBINATORICS

1. INTRODUCTION TO PERMUTATIONS AND COMBINATIONS

1. INTRODUCTION

$$\Rightarrow n! \text{ (or) } n$$

$$\Rightarrow n! = 1 \cdot 2 \cdot 3 \cdots n \quad \left. \begin{array}{l} \Rightarrow 0! = 1 \\ \Rightarrow \text{factorial of Negative numbers does not exist.} \end{array} \right\}$$

$$\Rightarrow \boxed{n! = (n-1)! \times n}$$

$$\begin{aligned} \Rightarrow (n+1)! - n! &\Rightarrow n!(n+1) - n! \\ &\Rightarrow n!(n+1-r) \end{aligned} \quad \left| \quad \begin{aligned} \Rightarrow (n-1)! + n! &= (n-1)! + (n-1)! \cdot n \\ &= (n-1)! (1+n) \end{aligned} \right.$$

$$\boxed{(n+1)! - n! = n \cdot (n!)} \quad \left| \quad \boxed{(n-1)! + n! = (n+1)(n-1)!} \right.$$

$$\begin{aligned} \Rightarrow n_{Pr} &= \frac{n!}{(n-r)!} \Rightarrow 5P_3 = \frac{5 \times 4 \times 3}{\underbrace{(3 \text{ literals})}} = 60 \\ &\Rightarrow 10P_4 = 10 \times 9 \times 8 \times 7 = 5040 \end{aligned} \quad \left| \quad \Rightarrow n_{Cr} = \frac{n!}{(n-r)! \cdot r!} \right.$$

$$\Rightarrow \boxed{n_{Pr} = n_{Cr} \times r!}$$

2. PROPERTIES OF NCB

$$\Rightarrow n_{Cr} = n_{C_{n-r}} \Rightarrow \text{let } n-r=x \Rightarrow n_{Cr} = n_{Cx} \quad \left. \begin{array}{l} \text{Properties of NCB-2} \\ \text{Properties of NCB-3} \end{array} \right\}$$

$$\begin{aligned} \frac{n!}{(n-r)! \cdot r!} &= \frac{n!}{(n-x)! \cdot x!} \\ &= \frac{n!}{(n-(n-r))! \cdot (n-r)!} \end{aligned}$$

$$\left. \begin{array}{l} \text{If } n_{Cx} = n_{Cy} \text{ then } \left\{ \begin{array}{l} x=y \text{ or} \\ y=n-x \text{ or} \\ x=n-y \end{array} \right. \end{array} \right\}$$

$$\frac{n!}{(n-r)! \cdot r!} = \frac{n!}{r! \cdot (n-r)!} \quad \therefore \boxed{n_{Cr} = n_{C_{n-r}}}$$

INTRODUCTION TO PERMUTATION AND COMBINATORICS

Now, consider n_{cr} Now the value of n_{cr} is more iff

a) n is even $\Rightarrow r = n/2$

b) n is odd $\Rightarrow r = \frac{n-1}{2}$ or $r = \frac{n+1}{2}$

c) $n_{cr} = n_{cy} \Rightarrow x+y=n$

3. PROPERTIES OF NCR--2

$$\Rightarrow n_{cr} + n_{c_{r-1}} = (n+1)_{c_r}$$

$$\Rightarrow \frac{n!}{(n-r)!r!} + \frac{n!}{(n-r+1)!(r-1)!}$$

$$= \frac{n!}{(n-r)! (r-1)! r} + \frac{n!}{(n-r)! (n-r+1) (r-1)!}$$

$$= \frac{n!}{(n-r)! (r-1)!} \left[\frac{1}{r} + \frac{1}{n-r+1} \right]$$

$$= \frac{n!}{(n-r)! (r-1)!} \left[\frac{n-r+1+r}{(n-r+1)r} \right]$$

$$= \frac{n! (n+1)}{(n-r+1)! r!} = \frac{(n+1)!}{(n+1-r)! r!} = (n+1)_{c_r}$$

Ex:

$$10_{c_4} + 10_{c_3} = 11_{c_4}$$

$$= 10_{c_3} + 10_{c_4} = ? \text{ we think } r=3 \text{ but } \times r=4$$

$$(10_{c_4} + 10_{c_3}) = 11_{c_4}$$

$$10_{c_4} + 10_{c_3} = (11)_{c_4} \text{ (1 more than } 10_{c_4} \text{)} \\ \text{largest of } 4, 3$$

4. PROPERTIES OF NCR-3

$$\Rightarrow n_{cr} = \frac{n}{r} (n-1)_{c_{r-1}}$$

$$\Rightarrow n_{cr} = \frac{n (n-1)!}{(n-r)! (r-1)! r}$$

$$= \frac{n}{r} \left[\frac{(n-1)!}{(n-r)! (r-1)!} \right]$$

$$= \frac{n}{r} \left[\frac{(n-1)!}{(n-1-r)! (r-1)!} \right] = \frac{n}{r} (n-1)_{c_{r-1}}$$

$$10_{c_3} = \frac{10!}{7! 3!} \Rightarrow \frac{7! \times 8 \times 9 \times 10}{7! \times 3 \times 2 \times 1} \\ 10_{c_3} = \frac{10}{3} \times 9_{c_2} = \frac{8 \times 9 \times 10}{3 \times 2 \times 1} \\ = \frac{10}{3} \times \frac{9}{2} \times 8_{c_1} = \frac{10}{3} \times \frac{9}{2} \times \frac{8}{1} \times 7_{c_0} = \frac{10 \times 9 \times 8}{3 \times 2 \times 1}$$

5. PROPERTIES

$$\Rightarrow \frac{n_{cr}}{n_{c_{r-1}}} =$$

$$\frac{n_{cr}}{n_{c_{r-1}}} =$$

$$\frac{n_{cr}}{n_{c_{r-1}}} =$$

6. Example

$$W.K.T = \frac{n}{r}$$

$$= \frac{21}{84}$$

$$= \frac{7r}{10r}$$

7. Example

$$90_{c_9} + \sum_{r=0}^{10}$$

$$= 90_{c_9} +$$

$$= n_{c_r} +$$

$$= 100_{c_8}$$

②

5. PROPERTIES OF NCR-4

$$\Rightarrow \frac{nCr}{nCr-1} = \frac{(n-r+1)}{r}$$

$$\frac{nCr}{nCr-1} = \frac{n!}{(n-r)!r!} \times \frac{(n-r+1)!}{n!} \times \frac{(r-1)!}{r!}$$

$$\frac{nCr}{nCr-1} = \frac{(n-r+1)}{r}$$

$$\begin{cases} nC_0 + nC_1 + nC_2 + \dots + nC_n = 2^n \\ nC_0 + nC_2 + nC_4 + \dots = 2^{n-1} \\ nC_1 + nC_3 + nC_5 + \dots = 2^{n-1} \end{cases}$$

$$\begin{cases} {}^{13}C_0 + {}^{13}C_1 + \dots + {}^{13}C_{13} = 2^{13} \\ {}^{13}C_0 + {}^{13}C_2 + {}^{13}C_4 + \dots = 2^{12} \\ {}^{13}C_1 + {}^{13}C_3 + {}^{13}C_5 + \dots = 2^{12} \end{cases}$$

6. EXAMPLE ONE NCR

W.K.T = $\frac{nCr}{nCr-1} = \frac{(n-r+1)}{r}$ | Given $nCr-1 = 36$, $nCr = 84$, $nCr+1 = 126$

What is 'n' and 'r'?

$$= \frac{84}{2693} = \frac{n-r+1}{r}$$

$$= 7r = 3n - 3r + 3$$

$$= 10r = 3n + 3 \quad \text{--- (1)}$$

Now, $\frac{nCr+1}{nCr} = \frac{n-(r+1)+1}{r+1}$

$$= \frac{n-r}{r+1} \therefore \frac{nCr+1}{nCr} = \frac{n-r}{r+1}$$

Now, $\frac{n-r}{r+1} = \frac{126}{84} = \frac{3}{2}$

$$\Rightarrow 2n - 2r = 3r + 3$$

$$\Rightarrow 2n - 3 = 5r \quad \text{--- (2)}$$

Solving (1) and (2) $\boxed{n=9}$ and $\boxed{r=3}$

7. EXAMPLE 2 ON NCR

$$90C_9 + \sum_{r=0}^{10} (100-r)C_8 = ?$$

$$= 90C_9 + (100C_8 + 99C_8 + 98C_8 + 97C_8 + 96C_8 + 95C_8 + 94C_8 + 93C_8 + 92C_8 + 91C_8 + 90C_8)$$

$$= nC_r + nCr = (n+1)Cr \Rightarrow 90C_9 + 90C_8 = 100C_9 = 91C_9 = 92C_9 = 93C_9 = 94C_9$$

$$= 100C_8 + 100C_9 = 101C_9$$

8. Example 3 ON NCR

$$50C_{20} + 3 \cdot 50C_{21} + 3 \cdot 50C_{22} + 50C_{23}$$

$$= 50C_{20} + 50C_{21} + 50C_{21} + 50C_{21} + 50C_{22} + 50C_{22} + 50C_{22} + 50C_{23} + 50C_{21}$$

This process doesn't work

$$= 51C_{21} + 51C_{22} + 51C_{22} + 51C_{22} + 50C_{23}$$

$$51C_{22} + 51C_{22} + 51C_{22} + 50C_{23} \Rightarrow 51C_{21} + 51C_{22} + 51C_{22} + 50C_{23} + 51C_{22}$$

$$= 50C_{20} + 3(50C_{21} + 50C_{22}) + 50C_{23}$$

$$= 50C_{20} + 3(51C_{22}) + 50C_{23}$$

$$= 50C_{20} + 51C_{22} + 51C_{22} + 51C_{22} + 50C_{23}$$

This combination does not work

$$\Rightarrow 50C_{20} + 50C_{21} + 2(50C_{21}) + 2(50C_{22}) + 50C_{22} + 50C_{23}$$

$$\Rightarrow 51C_{21} + 2(50C_{21}) + 2(50C_{22}) + 51C_{23}$$

$$= 51C_{21} + 2(51C_{22}) + 51C_{23}$$

$$= 51C_{21} + 51C_{22} + 51C_{22} + 51C_{23}$$

$$= 51C_{22} + 51C_{23} = 53C_{23}$$

9. Example 4 ON NCR

$1! + 4! + 7! + 10! + \dots + 301!$ what is the last digit of the sum

and 2 last digits of sum?

$$\text{Sol: } 1! + 4! + 7! + 10! + \dots + 301!$$

$\Rightarrow$ Any thing after $10!$ will contain units digit as '0' and '00' as last 2 digits.

$\Rightarrow$ Now consider $5!$ to $7!$ it also contains unit digit as '0' because $5! = 120$

$\Rightarrow$ Now, the only thing that affects the sum is $1! + 4! = 25$

$\therefore$ Last digit in sum = 5

$$1 + 24 + 5040 = 5065$$

$\therefore$ Last 2 digits = 65

10. FUND

Assume +

3 Red ball

3 Black ball

$\Rightarrow$ This pr

2) Assum

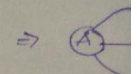
(a,b)

2 Shirts

2 pants

(1,2)

$\Rightarrow$ This pr



$\Rightarrow$ 10 Bl

10 Bl

11. Row

$\Rightarrow$ permo

$\Rightarrow$ choo

$\Rightarrow$ The

$\Rightarrow$ The

10 FUNDAMENTAL RULES OF COUNTING

Assume there are

3 Red balls of diff sizes } The no. of ways you can choose a ball out of
3 Black balls of diff sizes } this 6 balls is '6' ways. This problem can be
subdivided into two parts as

1) The no. of ways of choosing Redball = 3w

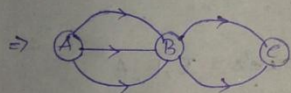
2) The no. of ways of choosing Blackball = $\frac{3w}{6w}$

⇒ This principle/Rule is called Addition principle of probability.

1) Assume there are

(a,b) } The no. of ways you can dress up is (a,1)
2 shirts } (a,2)
2 pants } (b,1)
(1,2) } (b,2) } 4 ways = (2 × 2)

⇒ This principle is called multiplication theorem on probability.



⇒ The no. of ways you can reach from 'A' to 'C' is
 $3 \times 2 = 6$ ways.

⇒ 10 Blue suits } The no. of ways you can dress up = $10 + 10 = 20$
10 Black suits }

11. ROW ARRANGEMENTS WITHOUT REPETITIONS

⇒ permutation = Arrangements

⇒ choosing = combination.

⇒ The no. of ways of arranging "abc" is abc, acb, bac, bca, cab, cba

⇒ The no. of ways of arranging 'n' things is $n!$

$$\begin{aligned} & \downarrow \downarrow \downarrow \downarrow \dots \downarrow \\ & n \quad (n-1) \quad (n-2) \quad (n-3) \dots 1 \\ & = 1 \cdot 2 \cdot 3 \dots (n) \\ & = n! \end{aligned}$$

12. EXAMPLES ON ROW ARRANGEMENTS WITHOUT REPETITIONS

1) CINEMA $\Rightarrow$ The no. of letters/arrangements of this word = $6! = 720$

Restricted Permutations $\Rightarrow$ The no. of words that are formed where the word always begin with 'C'.

$$\therefore \begin{array}{|c|c|c|c|c|c|} \hline C & & & & & \\ \hline \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 5 & 4 & 3 & 2 & 1 & \\ \hline \end{array} \therefore 1 \times 2 \times 3 \times 4 \times 5 = 120 \text{ words.}$$

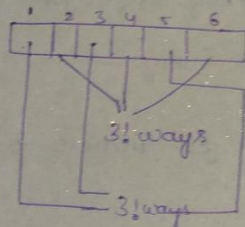
$\Rightarrow$ Start with 'C' and end with 'A'.

$$\begin{array}{|c|c|c|c|c|c|} \hline C & & & & & A \\ \hline \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 4 & 3 & 2 & 1 & & \\ \hline \end{array} = 4 \times 3 \times 2 \times 1 = 4! = 24.$$

$\Rightarrow$ 'C' and 'A' should be the Extremes.

$$\begin{array}{|c|c|c|c|c|c|} \hline C & & & & & A \\ \hline \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 4 & 3 & 2 & 1 & & \\ \hline \end{array} \quad \& \quad \begin{array}{|c|c|c|c|c|c|} \hline A & & & & & C \\ \hline \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 4 & 3 & 2 & 1 & & \\ \hline \end{array} = 4! + 4! = 48 \text{ words.}$$

$\Rightarrow$ Vowels are at even positions

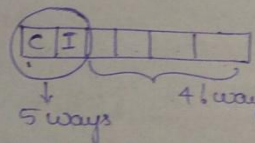


Vowels = I, E, A

Consonants = C, N, M

$$= 3! \times 3! = 36 \text{ ways.}$$

$\Rightarrow$ 'C' and 'I' should always be together



$$\begin{aligned} &= 5 \times 4! \times 2! \text{ (Internal permutations of CI)} \\ &= 5 \times 24 \times 2 \\ &= 10 \times 24 \\ &= 240 \end{aligned}$$

(CI)NEMA

$$\begin{aligned} \text{Single unit} &= 5! \times 2! \\ &= 240 \end{aligned}$$

Arranged

The no. of

are identical

$\Rightarrow$ we have

kind

them

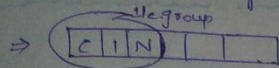
1208 (6)

$1! = 720$

always

words.

⇒ C I N should always be together



$4! \times 3! = 24 \times 6 = 124$
 4 groups
 $= 4! \text{ ways}$
 ↳ Internal permutations

⇒ 'C' and 'I' should never be together = Total Arrangements - CI come together

$192 \leftarrow 4 \times 2! \times 4! \leftarrow$ [C I _ _ _]

$144 \leftarrow 3 \times 2! \times 4! \leftarrow$ [C _ I _ _]

$96 \leftarrow 2 \times 2! \times 4! \leftarrow$ [C _ _ I _]

$48 \leftarrow 1 \times 2! \times 4! \leftarrow$ [C _ _ _ I]

480

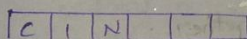
$\Rightarrow 6! - (5! \times 2!) = 480$

↓
 This procedure is applicable only for 2 letters not for 3 Alphabets.

(4!)

words.

⇒ C I N should always be together and 'S' should always be 'O' and 'N'.



$2 \times 4 \times 1 \times 3! = 4 \times 6 = 24 \text{ words} \times 2$ (CIN can be changed).
 $= 48 \text{ words}$

13. ROW ARRANGEMENTS WITH REPETITIONS

Arrangements with Repetition (some objects are identical)

The no. of ways of arranging 'n' elements in a row in which 'r' elements are identical is called $= \frac{n!}{r!}$

⇒ we have 'n' objects out of which 'm' are of one kind, 'r' are of another kind and 'p' objects are of other kind, then the no. of ways to arrange

them is $\frac{n!}{m! \times r! \times p!}$

14. EXAMPLES ON ROW ARRANGEMENTS WITH REPETITIONS

RAVINDRABABU $\Rightarrow$ NO. of 12 lettered words that can be formed is

R - 2 times
A - 3 times
B - 2 times

$$\frac{12!}{2! 3! 2!}$$

Restricted Permutations $\Rightarrow$ start with R

[R] [] [] [] [] [] [] [] [] [] [] []

$$\frac{11!}{3! 2!}$$

$\Rightarrow$ start with R and end with U

[R] [] [] [] [] [] [] [] [] [] [U]

$$\frac{10!}{3! 2!}$$

$\Rightarrow$ Vowels and consonants should come together.

Vowels = A, I, A, A, U

Consonants = R, V, N, D, R, B, B.

} 2 Groups 2! ways

$$\frac{10!}{3! 2!} \times \frac{5!}{3!} \times \frac{7!}{2! 2!}$$

15. EXAMPLES ON ROW ARRANGEMENTS WITH REPETITIONS - 2

RAVINDRABABU

R - 2 times

B - 2 times

A - 3 times

$\Rightarrow$ All A's should come together

[AAA] [] [] [] [] [] [] [] [] [] [] []

$$= \frac{10!}{2! 2!} \times \frac{3!}{3!} = \frac{10!}{2! 2!} = \binom{AAABBB}{AAA, B-B}$$

$\Rightarrow$ All A's should be together and B's should not be together

[AAA] [] [] [] [] [] [] [] [] [] [] []

=(All A's coming together) - (All B's not coming together)

$$= \left(\frac{10!}{2! 2!} - \frac{9!}{2!} \times \frac{3!}{3!} \times \frac{2!}{2!} \right) [AAA][BB] R, V, N, D, R, U$$

16. EXAMPLES ON

R's $\Rightarrow$ not together

B's $\Rightarrow$ should not be

Represent together on B's

$$\frac{12!}{2! 3! 2!}$$

17. Example

2, 3, 4, 4, 5

{ 8 }

18. Example

2, 3, 4, 4, 5

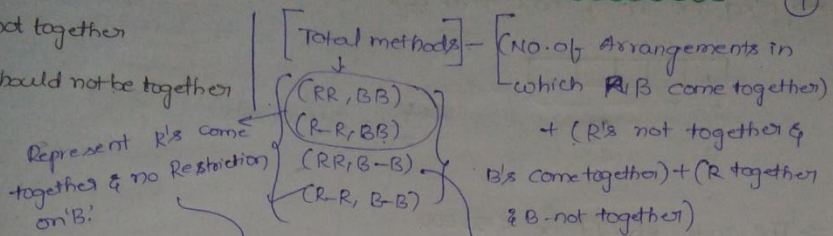
$\Rightarrow$ Even die

IONS-1
⑧

16. EXAMPLES ON ROW ARRANGEMENTS WITH REPETITIONS-3

R's → not together

B's → should not be together



$$\therefore \frac{12!}{2!3!2!} - \left[\frac{11! \times 2!}{2!3!2!} + \left(\frac{10!}{3!2!} \right) \right] \rightarrow \text{Total} - (R \text{ come together})$$

$$\left(\frac{11!}{3! \times 2!} - \frac{10!}{3!} \right)$$

$$\therefore \text{The No. of ways} = \frac{12!}{2!3!2!} - \left[\frac{11! \times 2!}{2!3!2!} + \left(\frac{11!}{3!2!} - \frac{10!}{3!} \right) \right]$$

17. EXAMPLES ON ROW ARRANGEMENTS WITH REPETITIONS-4

2, 3, 4, 4, 5

$$\Rightarrow \text{Total no. of Nds} = \frac{5!}{2!} = 60$$

$$\Rightarrow \text{Even nds} = \frac{4!}{2!} = 12 \quad \left| \quad \frac{4!}{1!} = 24 \quad (2, 4, 4)$$

$$\Rightarrow \underline{36} \text{ even nds.}$$

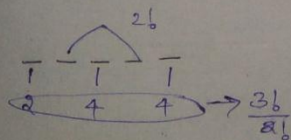
$$\text{= odd nos: } \frac{4!}{2!} \quad \left| \quad \frac{4!}{2!} \quad 5$$

$$= 2 \times \frac{4!}{2!} = 24 \text{ odd nds.}$$

18. EXAMPLES ON ROW ARRANGEMENTS WITH REPETITIONS-5

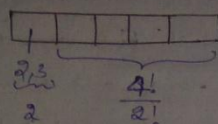
2, 3, 4, 4, 5

⇒ Even digits should be at odd places. (2, 4, 4)



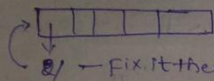
$$\therefore \frac{3!}{2!} \times 2! = 6 \text{ ways.}$$

⇒ Find the no. of nos. that are $< 40,000$.



$$= \frac{2 \times 4!}{2!} = 24 \text{ words}$$

⇒ The nos. should be $< 50,000$



Now, $\frac{4!}{2!}$ nos

$$= 2 \left(\frac{4!}{2!} \right) + 4!$$

3 - Fix it then → Now, $\frac{4!}{2!}$ nos

4 - Fix it then → Now, $\frac{4!}{2!}$ nos

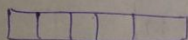
$$= 48$$

19. EXAMPLES ON ROW ARRANGEMENTS WITH REPETITIONS-6

2, 3, 4, 4, 5

⇒ Find the 5 digit nos. which are $\div$ by 2. = 36 even nos.

⇒ $\div$ by 3 ⇒ (A no. is $\div$ by 3 if the total sum is $\div$ by 3)



$$2+3+4+4+5=18$$

$$\therefore \text{The nos. are } = \frac{5!}{2!}$$

⇒ Divisible by 4 (last 2 digits should be $\div$ by 4)

$$= \frac{4!}{2!} = 3!$$

$$= \frac{4!}{2!} = 3!$$

$$= \frac{3!}{2!} = 3!$$

$$= \frac{5!}{2!} = 3!$$

$$\therefore \text{The total nos.} = 3! + 3! + 2 \left(\frac{3!}{2!} \right)$$

$$= 3(3!)$$

$$= 18$$

⇒ How many

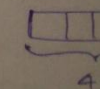
⇒ $\div$ by 5

$$\frac{4!}{2!} \times 1$$

$$\therefore \text{The no. of words} = 12$$

20. Example

0, 2, 3, 4, 5 ⇒



$$\frac{(2 \times 4!)}{3}$$

(10)

20. EXAMPLES ON ROW ARRANGEMENTS WITH REPETITIONS0, 2, 3, 4, 5 $\Rightarrow$ Find no. of 5 digit Number

(11)

$$\Rightarrow \boxed{} \boxed{} \boxed{} \boxed{} \boxed{} = (\text{Total Nos}) - (\text{Nos having '0' in first place})$$

$$= 5! - 4!$$

$$= \underline{96}$$

(or)

$$\begin{array}{c} \downarrow \\ \boxed{1} \boxed{} \boxed{} \boxed{} \boxed{} \\ \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \\ (2,3,4,5) \quad 4! \\ 4 \end{array} = 4 \times 4! \text{ nos}$$

$$= \underline{96 \text{ nos}}$$

 $\Rightarrow$ Find the total no. of 5 digit nos that are even?

$$= \boxed{} \boxed{} \boxed{} \boxed{} \boxed{}$$

$$(0,2,4) = 3 \text{ chances}$$

Now, $\boxed{} \boxed{} \boxed{} \boxed{} \boxed{0}$ Nos = $4! \text{ nos}$

$\boxed{} \boxed{} \boxed{} \boxed{} \boxed{2}$ $\downarrow \quad \downarrow \quad \downarrow \quad \downarrow$ $(3,4,5) \quad 3!$ $\downarrow$ 3

$$\boxed{\text{nos} = 3 \times 3!}$$

$\boxed{} \boxed{} \boxed{} \boxed{} \boxed{4}$ $\downarrow \quad \downarrow \quad \downarrow \quad \downarrow$ $(2,3,5) \quad 3!$ $\downarrow$ 3

$$= 3 \times 3! \text{ Nos}$$

$$\therefore \text{Total Nos} = 4! + (3 \times 3!) + (3 \times 3!)$$

$$= 24 + 3(6) + 3(6)$$

$$= 24 + 36$$

$$= \underline{60}$$

$$2(3!/2!)$$

 $\Rightarrow$ How many 5 digit nos are present that are divisible by 5?

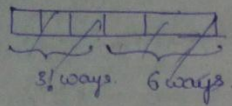
$$\boxed{} \boxed{} \boxed{} \boxed{} \boxed{0} = 4! \text{ Nos}$$

$$\Rightarrow \boxed{\text{Total Nos} = 4! + 3 \times 3!}$$

$\boxed{} \boxed{} \boxed{} \boxed{} \boxed{5} = 3 \times 3! \text{ Nos}$

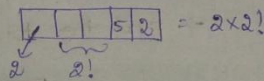
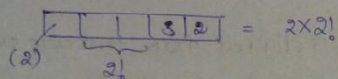
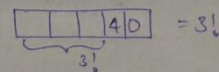
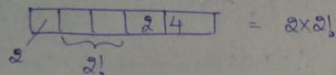
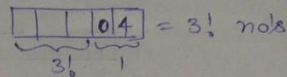
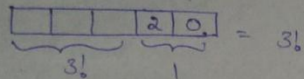
$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow$ $(2,3,4) \quad 3! \text{ ways}$ $\downarrow$ 3

⇒ find no. of 5 digit nos that are divisible by 4.



Given nos = 0, 2, 3, 4, 5

(20) (40)
(24) (52)
(32)
(04)



∴ Total nos divisible by 4 = $3(3!) + 3(2 \times 2!)$

$$= 3(6) + 3(4) \\ = 18 + 12 = \underline{30}$$

21. RANK IN A DICTIONARY

⇒ RAVI - 4! words can be formed. (Find the rank of RAVI)

⇒ AIRV - Alphabetical order

⇒ A - - - = 3! words

I - - - = 3! words

R A I V = 1! word

R A V I = 1 word

14

⇒ RANK (Find the rank of the word Rank)

AKNR

A - - - = 3!

K - - - = 3!

N - - - = 3!

~~R - - - = 3!~~

R A K - - = 1! word

R A I N K - 1 word

20

∴ Rank of the word

RANK = 20

⇒ Find the Rank

⇒ Alphabetical order

A -

O -

R -

S A

S O

S C

S I

S

S

⇒ find the

Alphabetical order

E - - -

G - - -

I E - -

I G E -

I G I -

I G N -

I G N -

I G N -

22. RANK

⇒ Given +

Now find

(12)

Find the Rank of Saurav. = 6! words are formed

⇒ Alphabetical order = A O R S U V

$$A - 5!$$

$$O - 5!$$

$$R - 5!$$

$$\boxed{S} A = 4!$$

$$\boxed{S} \boxed{O} A = 3!$$

$$\boxed{S} \boxed{O} R = 3!$$

$$\boxed{S} \boxed{O} \boxed{U} A = 2!$$

$$\boxed{S} \boxed{O} \boxed{O} \boxed{R} \boxed{A} \boxed{V} = 1$$

399

⇒ find the Rank of IGNITE

Alphabetical order = E [✓]G [✓]I [✓]I [✓]N T

$$E - - - - - = 5! / 2! = 60$$

$$G - - - - - = 5! / 2! = 60$$

$$\boxed{I} E - - - - = 4! = 24$$

$$\boxed{I} \boxed{G} E - - - = 3! = 6$$

$$\boxed{I} \boxed{G} I - - - = 3! = 6$$

$$\boxed{I} \boxed{G} \boxed{N} E - - = 2! = 2$$

$$\boxed{I} \boxed{G} \boxed{N} \boxed{I} E T - 2! = 2$$

$$\boxed{I} \boxed{G} \boxed{N} \boxed{I} \boxed{T} E - 1$$

160

22. RANK IN A DICTIONARY-2

⇒ Given the Alphabets A, I, R, V Find the word whose rank is 16

Now find the word whose rank is 14?

$A \text{ ---} = 3! = 6 (1-6)$
 $I \text{ ---} = 3! = 6 (7-12)$
 $R A \text{ ---} = 2! = 2 (13-14)$

$RAIV = 13$

$RAVI = 14^{th}$

⇒ find the word whose rank is 20. Rank = 18

$A \text{ ---} = 3! = 6 (1-6)$
 $I \text{ ---} = 3! = 6 (7-12)$
 $R \text{ ---} = 3! = 6 (13-18)$
 $V A \text{ ---} = 2! = 2 (19-20)$

$VARI = 19$

$VARI = 20$

$A \text{ ---} = 3! = 6$

$I \text{ ---} = 3! = 6$

$R \text{ ---} = 2! = 2$

$R I \text{ ---} = 2$

$R V \text{ ---} = 2$

$R V A I = 17$

$R V I A = 18$

$R V I A = 18$

⇒ find the word whose rank is 160. (E, G, I, N, T)

$E \text{ ---} = 5! = 120 (1-120)$

$G \text{ ---} = 4! = 24 (121-144)$

$I E \text{ ---} = 3! = 6 (145-150)$

$X \text{ } I G \text{ ---} = 2! = 2 (151-152)$
 $I G E \text{ ---} = 2! = 2 (153-154)$
 $I G I \text{ ---} = 2! = 2 (155-156)$

$I G N \text{ ---} = 2! = 2 (157-158)$

$I G N E \text{ ---} = 1! = 1 (159-160)$

$I G N T E = 160$

23) CIRCULAR

⇒ The no. of

a, b, c

abc

acb

bac

bca

cab

cba

24) CIRCULAR

How many

a circle?

RAVINDRA

CIRCULAR

25) Example

A family

how many w

and father

(14)

23) CIRCULAR PERMUTATIONS WITHOUT REPETITIONS.

→ The no. of arrangements of 'n' distinct objects around a circle is $(n-1)!$.

abc

abc

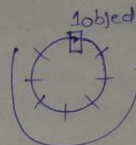
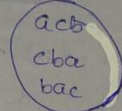
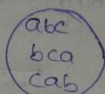
acb

bac

bca

cab

cba

 $(n-1)$ objects $(n-1)!$ ways.

∴ or permutations of 'n' distinct objects = $(n-1)!$

(15)

24. CIRCULAR PERMUTATIONS WITH REPETITIONS

IA = 18

How many ways you can arrange letters of the word RAVINDRA? In a circle?

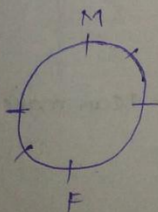
RAVINDRA = 8 letters $\Rightarrow$ No. of Arrangements = $\frac{8!}{2! \cdot 2!}$ / A/R are repeated twice

CIRCULAR : $\frac{8!}{2! \cdot 2!}$

PERMUTATIONS : $\frac{12!}{2!}$

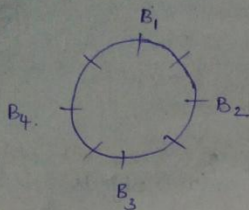
25. EXAMPLES ON CIRCULAR PERMUTATIONS -- I

A family has 6 mem, Mother, Father, sons (S_1, S_2), Daughters (D_1, D_2). Now how many ways you can make them sit in a table such that Mother and Father faces each other.



Then 4 mem are remaining and 4 objects are present $\therefore 4!$ ways.

26. We have 4 Boys = B_1, B_2, B_3, B_4 } what are the no. of ways in which no two boys are together and no two girls are together.
 4 Girls = G_1, G_2, G_3, G_4



$\Rightarrow$ 4 boys can be arranged in circle in $3!$ ways

$\Rightarrow$ In the remaining 4 blanks 4 girls can be seated in $4!$ ways

$$\therefore \text{No. of ways} = 4! \times 3!$$

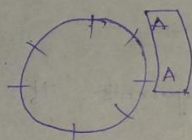
$\Rightarrow$ No. of ways in which no boy and Girl are together is $7! - (4! \times 2!)$

26. EXAMPLE ON CIRCULAR PERMUTATIONS -- 2

RAVINDRA $\Rightarrow$ arrange in circle and ① Two a's always come together

$$\downarrow$$

$$\frac{8!}{2!2!} \text{ ways.}$$



$$= \frac{6!}{2!}$$

R V I N D R A A

7 objects

6! ways

R is repeated two times.

$\Rightarrow$ Two a's should never come together.

$$\text{No. 2 a's come together} = \left(\frac{8!}{2!2!} \right) - \left(\frac{6!}{2!} \right)$$

27. EXAMPLES ON GARLAND MODEL QUESTIONS

The no. of ways in which we can make garland from 'n' flowers is $\frac{(n-1)!}{2}$

There are 5 flowers. Now the no. of ways that I can make garland

$$\text{is } \frac{(n-1)!}{2} = \frac{(5-1)!}{2} = \frac{4!}{2} = \frac{12}{2} = 6$$

28. INTRODU

The no. of w

is ' n_c ' =

$$3_c = 3 =$$

29. EXAMP

10 Boys, 8 G

group sho

$$= 10_c \times 8_c$$

= Majority o

$\Rightarrow$ Majority o

$\Rightarrow$ We have

2 wicket

28. INTRODUCTION TO COMBINATIONS

The no. of ways of choosing 'r' objects from a set of 'n' distinct objects is

$${}^nC_r = \frac{n!}{(n-r)!r!}$$

$${}^3C_3 = 3 = \begin{matrix} abc \\ \left\{ \begin{matrix} ab \\ bc \\ ac \end{matrix} \right\} \end{matrix} \text{ 3 ways.}$$

$${}^5C_3 = 10 = \begin{matrix} abcde \\ \left\{ \begin{matrix} ab, ac, ad, ae \\ bc, bd, be \\ cd, ce, de \end{matrix} \right\} \end{matrix}$$

29. EXAMPLE ON COMBINATION--1

10 Boys, 8 Girls. In how many ways you can choose 5 of them such that the group should contain 3 Boys and 2 Girls?

$$= {}^{10}C_3 \times {}^8C_2$$

$$= \text{Majority are Boys} \Rightarrow (3B+2G) + (4B+1G) + (5B+0G)$$

$$= ({}^{10}C_3 + {}^8C_2) + ({}^{10}C_4 + {}^8C_1) + ({}^{10}C_5)$$

$$\Rightarrow \text{Majority are Girls} = (3G+2B) + (4G+1B) + (5G+0B)$$

$$= ({}^8C_3 + {}^{10}C_2) + ({}^8C_4 + {}^{10}C_1) + ({}^8C_5)$$

$\Rightarrow$ We have 22 players, form a team of 11, we have 12 Batsman, 8 Bowlers, 2 wicket keepers, the team should contain (5 Batsmen, 5 bowlers, 1 wk) atleast.

12	8	2wk	ways
BATSMAN	BOWLERS		
${}^{12}C_5$	${}^8C_5$	${}^2C_1$	${}^{12}C_5 \times {}^8C_5 \times {}^2C_1$
${}^{12}C_6$	${}^8C_4$	${}^2C_1$	${}^{12}C_6 \times {}^8C_4 \times {}^2C_1$
${}^{12}C_6$	${}^8C_5$	1	${}^{12}C_6 \times {}^8C_5 \times 1$
:	:	:	:

30. EXAMPLE ON COMBINATIONS

⇒ A mathematics question paper contains 12 Questions and we are supposed to answer 8. Then how many ways you can answer the question paper? ${}^{12}C_8$.

⇒ The QP has 2 parts, the first 2 parts contains questions from (1-6) and 2nd part contains questions from (7-12). The Restriction is you have to answer atleast 3 from part 1 and in total you have to answer 8 questions?

The no. of ways are,

$$\begin{array}{l} \text{PART 1} \quad (1 \ 2 \ 3 \ 4 \ 5 \ 6) \quad \text{PART 2} \quad (7 \ 8 \ 9 \ 10 \ 11 \ 12) \\ \Rightarrow {}^6C_3 \times {}^6C_5 = \\ \Rightarrow {}^6C_4 \times {}^6C_4 = \\ \Rightarrow {}^6C_5 \times {}^6C_3 = \\ \Rightarrow {}^6C_6 \times {}^6C_2 = \end{array} \left\{ \begin{array}{l} \text{The total ways are} \\ ({}^6C_3 \times {}^6C_5) + ({}^6C_4 \times {}^6C_4) \\ + ({}^6C_5 \times {}^6C_3) + ({}^6C_6 \times {}^6C_2) \end{array} \right.$$

31. EXAMPLES ON COMBINATIONS

There are 10 couples (10m, 10w). Now, I want to form a team of 4 mem such that I have (2m and 2w) then how many ways we can form it.

No. of ways ${}^{10}C_2 \times {}^{10}C_2$

⇒ Now In the team of 4 people I should see that no couple is included.

⇒ ${}^{10}C_2 \times {}^8C_2$

⇒ Now In the 4 mem team we need to include exactly one couple then

⇒ Now choose a couple from the 10 couples = ${}^{10}C_1$ (I have included 2 mem since

⇒ Now, among the 9m and 9w, I have chosen a couple) choose 1 mem = ${}^9C_1$

⇒ Since I need to include exactly one couple the I should not include the wife of the person chosen above, so I'm left with 8 mem, so choose 1 from them = ${}^8C_1$ ways.

⇒ Now In the 4 selected.

case 1: +1, left

case 2: +1,

⇒ If H_1 is not so may or may not

ways =

∴ No. of u

u

⇒ w_2, w_3 show

w_2	w_3
✓	✓
✓	×
×	✓
×	×

32. EXAMPLES

⇒ There are 10 have share

⇒ There are 2 persons =

(18)

$$\therefore \text{The total no. of ways} = {}^{10}C_2 \times {}^9C_1 \times {}^8C_1$$

(19)

$\Rightarrow$ Now, In the 4 member team, If H_1 is selected then W_5 should be selected.

Case 1: H_1 is selected $\Rightarrow W_5$ should be included, Now, 9 men, 9 women left $\Rightarrow$ No. of ways of selecting = ${}^9C_1 \times {}^9C_1$

Case 2: H_1 is not included then, among 9 men choose 2 = ${}^9C_2$ ways.

$\Rightarrow$ If H_1 is not selected it doesn't mean that you should not select W_5 , it may or may not be selected. $\Rightarrow {}^{10}C_2$

$$\text{ways} = {}^9C_2 \times {}^{10}C_2$$

$\therefore$ No. of ways = Case 1 + Case 2

$$\text{ways} = {}^9C_1 \times {}^9C_1 + {}^9C_2 \times {}^{10}C_2$$

$\Rightarrow W_2, W_3$ should not be selected simultaneously.

$$\begin{array}{cc} W_2 & W_3 \\ \hline \checkmark & \checkmark \end{array} = {}^{10}C_2$$

$$\checkmark \quad \times = {}^{10}C_2 \times {}^8C_1$$

$$\times \quad \checkmark = {}^{10}C_2 \times {}^8C_1$$

$$\times \quad \times = {}^{10}C_2 \times {}^8C_2$$

These are the cases that we need

$$= {}^{10}C_2 ({}^8C_1 + {}^8C_1 + {}^8C_2)$$

ST. ORDER

32. EXAMPLES ON COMBINATIONS - 4

$\Rightarrow$ There are 10 couples (10m, 10w) How many different ways we can have shakehands between them?

$\Rightarrow$ There are 20 people and we can have shakehands between any two persons = $\therefore$ The total no. of shake hands = ${}^{20}C_2$

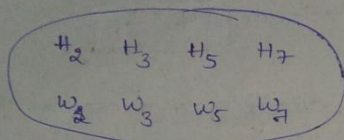
$$= \frac{20 \times 19}{2} = 190 \text{ Shake Hands.}$$

⇒ find the no. of shake hands that doesnot include a couple

$$\text{No. of shake hands} = \binom{20}{2} - 10$$

No. of shake hands of couples = 10
10-couples = 10-shakehands

⇒ Let us say $H_1, H_2, H_3, \dots, H_{10}$
 $w_1, w_2, w_3, \dots, w_{10}$ } The condition is that there should not be hand shake between prime nos.



There should not be shakehands between these problems.

∴ The No. of shakehands = $\binom{20}{2} - 8$

33. REPETITIONS IN COMBINATIONS

$2ab = \frac{3!}{2!} = 3 \text{ ngs}$

$abcd = \frac{5!}{2!}$

$--- (3C_2 \times 1)$

$= (3 \text{ ways})$

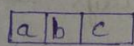
$= \frac{5!}{2!} \times 3! \text{ (take two objects and fix them)}$

and remaining objects can be placed in 3! ways.

There are 'n' numbers/objects in which 'r' are of similar kind then the no. of words that can be formed = $\frac{n!}{r!} = \binom{n}{r} (n-r)!$ → Arrange the remaining ones
→ choose the repeating objects

34. ORDER

⇒ we have abc, have to form all 3 lettered words such that a should occur before b. $a \bar{b}, a-b$ → 1 letter between a and b.



3 positions

c a b -

a c b -

a b c -

3 ways.

⇒ 1s choose 2 positions from available 3 positions = $\frac{3!}{2!} = 3$

$\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \begin{matrix} abc \\ acb \\ cab \end{matrix}$

⇒ PERMUTATION

in Alphabet

vowels =

⇒ Now choose and place

⇒ Remaining

⇒ We have

'n' lettered words

of words = n!

35. EXAMPLE

RAVIND →

R should

⇒ Fix (R, A)

⇒ R should

R → A → V

⇒ R should

(20)

⇒ PERMUTATIONS (The condition is that vowels should always be in Alphabetical order)

vowels = (AEIOU) = This is the order in which vowels should occur

⇒ Now choose 5 positions among 11 positions = ${}^{11}C_5$ ways (vowels and place the vowels.

cannot be permuted because order is specified here)

⇒ Remaining 'S' letters can be placed in $\frac{6!}{2!}$ ways (T Repeated 2 times)

$$\therefore \text{No of ways} = \frac{11!}{5!2!} \times \frac{6!}{2!}$$

$$\boxed{\text{No of ways} = \frac{11!}{5!2!}}$$

⇒ We have 'n' different letters and we are supposed to find all 'n' lettered words in which 'r' letters should be in some order then no. of words = $\frac{n!}{r!}$

35. EXAMPLE ON ORDER

RAVIND ⇒ The no. of possible words that can be formed in which

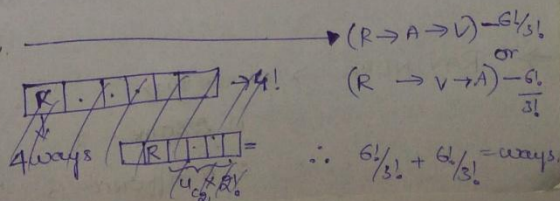
'R' should always occur before 'A'?

⇒ Fix (R, A) ⇒ out of 6 positions choose any two = ${}^6C_2 \times 4!$ Remaining words.
 $= \frac{6!}{2!4!} \times 4! = \frac{6!}{2!}$

⇒ R should come before 'A' and 'A' should come before 'V'.

R → A → V ⇒ ${}^6C_3 \times 3!$ Remaining words arrangements.

⇒ R should come before A and V.



36. EXAMPLES ON ORDER-2

RAVINDR → find all words such that 'R' occurs before A or 'R' occurs before 'B'.

⇒ find the total words and subtract the ways in which 'R' and ~~RA~~ occur before R. ($A \rightarrow V \rightarrow R, V \rightarrow A \rightarrow R$)

$$= 6! - ((6_3 \times 3!) + (6_3 \times 3!))$$

37. SELECTIONS WITH REPETITIONS

$$abcd = 4C_2 = 6$$

= ab
ac
ad
bc
bd
cd

$$aabc = \frac{4!}{2!} = 12$$

aa
ab
ac
bc

$$4 = 4C_2$$

38. EXAMPLE ON SELECTIONS WITH REPETITIONS

⇒ aabc ⇒ find/choose two objects

Case 1: Two letters are same — oneway = $2C_2$

Case 2: Two letters are different = $3C_2 = 3$ ways = (a, b, c)

Total =

$$2 + 3 = 5 \text{ ways}$$

⇒ RAVINDR ⇒ Choose two letters from this name

R-2
A-1
V-1
I-1
N-1
D-1

Case 1: Both the letters are same = 1 way.

Case 2: Both the letters are different = $6C_2$

Total =

$$6C_2 + 1 \text{ way}$$

⇒ choose 3 letters from the given letters.

Case 1: 3 different letters = $6C_3$

Case 2: 2 same + 1 different = $1 \times 5C_1$

$$= (6C_3 + 5) \text{ ways}$$

⇒ RAVINDRA ⇒ choose two

1) 2 same — 2

2) 2 different — $6C_2$

$$\text{Total} = (2 + 6C_2)$$

(22)

$$m_p = n_0 \times 2^p$$

1. INTRODUCTION

$$T(n) = T(n-1) +$$

$$T(n) = aT(n/b)$$

2. RECURRANCE

Sequence: Any

$$T(n)$$

$$n$$

$$T(0)$$

Recurrence Relation

present the
of previous

1.

Seq: 1, 3, 5

$$n = 0, 1, 2$$

$$T(0) = 1, T(1) =$$

$$T(n) =$$

(22)

before 'B'?

nd ~~the~~ V.

$$m_p = n_g \times n!$$

(23)

= 4/8

2. RECURRANCE RELATIONS

1. INTRODUCTION

$$T(n) = T(n-1) + T(n-2) + T(n-3) + K.$$

$$T(n) = aT(n/b) + K \rightarrow \text{masters theorem}$$

2. RECURRANCE RELATION AND SOLUTION

Sequence: Any order in which you write a number.

$$T(n) = 0, 2, 4, 6, 8, 10, \dots$$

$$n = 0, 1, 2, 3, 4, 5$$

$$T(0)=0, T(1)=2, T(2)=4, T(3)=6.$$

Recurrence Relation: It is a relation, in which we are possible to write present the n th term / Any term in the sequence as an mathematic Expression of previous terms then it is called RR.

Total =
= 4 ways

11/4/21

1/9

ad =

+ 1 way

ways

$$T(n) = T(n-1) + 2 \quad [n \geq 1] \quad \text{[for above example]}$$

The main aim of RR is to find such sol to the RRs.

$$\left. \begin{array}{l} T(0)=0 \\ T(1)=2 \end{array} \right\} \text{Initial conditions}$$

↓

Sol to above RR $T(n) = 2n$.

$$\text{LHS of RR} = T(n) = 2n$$

$$\begin{aligned} \text{RHS of RR} &= T(n-1) + 2 = 2(n-1) + 2 \\ &= 2n // \end{aligned}$$

Seq: 1, 3, 5, 7, 9, 11, ...

$$n = 0, 1, 2, 3, 4, 5$$

$$T(0)=1, T(1)=3, T(2)=5$$

$$T(n+1) + 2 = T(n)$$

$$T(n) = 2n + 1 \quad n \geq 1$$

$$T(0) = 1 \text{ (Initial condition)}$$

$$T(n) = 2n + 1$$

$$\text{LHS of RR} = T(n) = 2n + 1$$

$$\text{RHS of RR} = T(n-1) + 2$$

$$= T(2n+1-1) + 2 = 2(n-1) + 1 + 2$$

$$= 2n + 1 //$$

Sequence: 1, 2, 4, 8, 16, ...

$n: 0, 1, 2, 3, 4$

$T(0)=1, T(1)=2, T(2)=4, T(3)=8$

$$T(n) = 2T(n-1), n > 0$$

$T(n)=0, n=0 \rightarrow$ [Initial condition]

Sol: $T(n) = 2^n$ (24)

LHS = $T(n) = 2^n$

RHS = $2T(n-1)$

$= 2 \cdot 2^{n-1}$

$= 2^n$ // LHS=RHS

INDUCTION

3. Examples

For the recurrence relation $T(n) = 2T(n-1) - T(n-2)$, $n \geq 2$, find whether the following are solutions?

a) $T(n) = 3n$ b) $T(n) = 2^n$ c) $T(n) = 5$

opvm, a) $T(n) = 3n$

$T(n) = 2T(n-1) - T(n-2)$

LHS = $T(n) = 3n$

RHS = $2T(n-1) - T(n-2)$

$= 2[3(n-1)] - [3(n-2)]$

$= 6(n-1) - [3n-6]$

$= 6n - 6 - 3n + 6 = 3n$ ✓ This is soln

(or)

$T(n) = 3n$

$T(0) = 0$

$T(1) = 3$

$T(2) = 6$

$T(3) = 9$

$T(2) = 2T(1) - T(0)$

$= 2 \times 3 - 0$

$= 6$ ✓

$T(3) = 2[T(2) - T(1)]$

$= 2(6 - 3)$

$= 12 - 6 = 6$

b) $T(n) = 2^n$

$T(0) = 1$

$T(1) = 2$

$T(2) = 4$

$T(3) = 8$

$T(2) = 2T(1) - T(0)$

$= 2(2) - 1$

Not equal

$3 \neq 4$

$\therefore 2^n$ is not solution.

c) $T(n) = 5$

$T(0) = 5$

$T(1) = 5$

$T(2) = 5$

$T(3) = 5$

$T(2) = 2T(1) - T(0) = (2 \times 5) - 5 = 5$

$T(3) = 2T(2) - T(1)$

$= 2 \times 5 - 5 = 5$

4. GATE 2004 QUES

The Recurrence relat

a) $2^{n+1} - n - 2$ b) 2^n

$T(1) = 1$

opA: $2^{1+1} - 1 - 2 = 1$

opB: $2^1 - 1 = 1$

$T(2) = 2T(1) + 2 \Rightarrow$

opA: $2^2 - 2 - 2 = 4$

opB: $2^2 - 2 = 2$

5. GATE 2002 QUES

The solution to the

a) 2^k b) $3^{k+1} - 1$

$T(1) = 1$

a) Now if the in Big Oh notat

opvm: $T(n)$

$T(1) = 1$

(24)

4. GATE 2004 QUESTION

The Recurrence relation

$$T(n) = 2T(n-1) + n, n \geq 2$$

$$T(1) = 1$$

Evaluates to

(25)

$$a) 2^{n+1} - n - 2 \quad b) 2^n - n \quad c) 2^{n+1} - 2n - 2 \quad d) 2^n + n$$

LHS=RHS

$$T(1) = 1$$

$$\text{opt: } 2^{1+1} - 1 - 2 = 1 \checkmark$$

$$c) 2^2 - 2 - 2 = 0 \times$$

$$\text{opt: } 2^1 - 1 = 1 \checkmark$$

$$d) 2^n + n = 2^1 + 1 = 3 \times$$

$$T(2) = 2T(1) + 2 \Rightarrow 2(1) + 2 = 4$$

$$\text{opt: } 2^3 - 2 - 2 = 4 \checkmark$$

$$\text{opt: } 2^2 - 2 = 2$$

$$\text{opt: } 2^{n+1} - n - 2$$

5. GATE 2002 QUESTION

The solution to the Recurrence Relation $T(2^k) = 3T(2^{k-1}) + 1$

$$T(1) = 1 \text{ is}$$

$$a) 2^k \checkmark (3^{k+1} - 1)/2$$

$$c) 3^{\log_2 k}$$

$$d) 2^{\log_3 k}$$

$$T(1) = 1$$

Now if the question is $T(2^k) = 3T(2^{k-1}) + 1$ and options are given in both notations then how to solve them is.

$$T(2^k) = 3T(2^{k-1}) + 1 \Rightarrow \text{let } 2^k = n$$

$$2^k/2 = n/2 \Rightarrow 2^{k-1} = n/2$$

$$T(n) = 3T(n/2) + 1$$

$$T(n) = aT(n/b) + O(n^k \log^p n)$$

$$a=3, b=2, k=0, p=0.$$

$$a > b^k \Rightarrow T(n) = O(n^{\log_b a})$$

$$\Rightarrow T(n) = O(n^{\log_2 3})$$

$$= O(2^k)^{\log_2 3} = O(2^{k \cdot \log_2 3}) = O(2^{\log_2 3^k})$$

$$T(n) = O(3^k)$$

$$\text{Ques: } T(2^k) = (3^{k+1} - 1)/2$$

$$T(1) = 1 \Rightarrow \text{put } k=0 \text{ in } T=1$$

$$= (3^1 - 1)/2 = 1 \checkmark$$

$$\text{LHS: } T(2^k) = (3^{k+1} - 1)/2$$

$$\text{RHS: } 3T(2^{k-1}) + 1$$

$$= 3 \left[\frac{3^k - 1}{2} \right] + 1 = \frac{3(3^k - 1)}{2} + 1 = \frac{3^{k+1} - 3 + 2}{2} = \frac{3^{k+1} - 1}{2}$$

Now, if the questions are given of this form $T(n) = 3T(\sqrt{n}) + 1$ then convert

it as $T(n) = 3T(\sqrt{n}) + 1$

let $n = 2^k \Rightarrow T(2^k) = 3T(2^{k/2}) + 1$

$\Rightarrow$ let $2^k = p$ then $T(p) = 3T(\sqrt{p}) + 1$

$\hookrightarrow$ Apply master theorem

6. GATE 2008 QUESTION

When $n = 2^{2k}$ for some $k \geq 0$ the recurrence relation $T(n) = \sqrt{2}T(n/2) + \sqrt{n}$ evaluates to

$T(1) = 1$

a) $\sqrt{n}(\log n + 1)$ b) $\sqrt{n} \log n$ c) $\sqrt{n} \log \sqrt{n}$ d) $n \log \sqrt{n}$

$T(1) = 1 \Rightarrow T(2) = \sqrt{2} + (1) + \sqrt{2}$
 $= 2\sqrt{2}$

opt: $\sqrt{n}(\log n + 1) = T(n)$

$T(1) = \sqrt{1}(\log 1 + 1) = 1 \checkmark$

$T(2) = \sqrt{2}(\log 2 + 1)$

$= \sqrt{2}(2) = 2\sqrt{2}$

8. GATE 2008 QUESTION

Let x_n denote the no. of binary strings of length n that contain no consecutive 0's. which of the following recurrence relation does x_n satisfy

a) $x_n = 2x_{n-1}$ ~~b) $x_n = x_{n-1} + 1$~~ c) $x_n = x_{n-2} + n$ ~~d) $x_n = x_{n-1} + x_{n-2}$~~

opt: a

x_1 = The no. of Binary strings that contain no consecutive ones = $\{1, 0\}$

$\Rightarrow x_1 = 2$

opt a: $x_1 = 2x_0 = 2$ ~~$x_1 = 2x_0 + 1$~~

opt B: $x_2 = x_1 + 1 = 2 + 1 = 3$ ~~$x_2 = x_1 + 2$~~

opt c: $x_2 = x_1 + 1 = x_2 = 2 + 1 = 3$ ~~$x_2 = x_1 + 2 = 4$~~

opt D: $x_2 = x_1 + x_0 = 2 + 1 = 3$ ~~$x_2 = x_1 + 2 = 4$~~

$x_2 = \{01, 10, 11\}$

$= 3$

$x_3 = \{5\}$

opt: $x_3 = x_2 + x_1$

$= 3 + 2$

$= 5$

SETS

1. SETS AND

Set: well

Ex: A =

S =

Null set: $\emptyset$

subset: $\subset$

of B. $\subset B$

Note: for

Proper subset

proper subset

Note

2. POWER

Denoted by

$\Rightarrow$ If A is

power set

Ex: A =

subset

$\Rightarrow$ If A is

26

$$c) a_{n-2} + 2a_{n-1} + 2^{n-2}$$

$$d) 2a_{n-2} + 2a_{n-1} + 2^{n-2}$$

OpA: $a_3 = 8$ (from side analysis)

$$a_n = a_{n-2} + a_{n-1} + 2^{n-2}$$

$$a_3 = a_1 + a_2 + 2 = 0 + 1 + 2 = 3. \checkmark$$

$$\text{опБ: } a_1 + 2a_2 + 2 = 4 \times$$

opc: $2a_1 + a_2 + 2' = 3 \checkmark$

Opd: $a_3 = 4x$

ODA

opc: $2 \times 1 + 3 + 4 = 9$

sati'st

sati'st

1

Seq:- 1, 1, 1, 1, - - -

$$\underline{GF} = 1 \cdot x^0 + 1 \cdot x^1 + 1 \cdot x^2 + 1 \cdot x^3 + \dots$$

$$GF = 1 + x + x^2 + x^3 + \dots$$

$$\Rightarrow \text{Seq: } 1, 1, 1 \quad = \quad 1 + x + x^2 = 1 + x + x^2 * \frac{(x-1)}{(x-1)} = \frac{(x^3-1)}{(x-1)} = \left[\frac{1-x^3}{1-x} \right]$$

$$1 \cdot x^0 + 1 \cdot x^1 + 1 \cdot x^2$$

3> 1, 1, 1, ... n 1's

GF: $1 + x + x^2 + \dots + x^n = \frac{x^{n+1} - 1}{x - 1} = \left[\frac{1 - x^{n+1}}{1 - x} \right]$

NOTE: QUESTION 28

GENERATING FUNCTIONS

1. INTRODUCTION

Let us suppose that a source of a sequence of numbers is the generating function of the sequence. If we know the generating function, we can take coefficients in the sequence and assign them weights. In the GF, for the above sequence

4. INTRODUCTION TO PROPOSITIONAL CALCULUS.

1. INTRODUCTION

Proposition: Any declarative statement with a truth value assigned to it is called proposition. Ex: My name is Ravindra Babu Ravula.

2) I am hungry [yes/no is truth value]

3) $x = y + 1$ [No truth value]

2. CONNECTIVES $\wedge$ AND $\vee$ AND $\neg$

Propositional variable: Variable used to denote a proposition is called pv.

Connectives: These are the things which connect more than one propositional variable (propositions).

p: My name is RBR

Negation: $\neg p$: My name is not RBR

: It is not the case that my name is not RBR.

$\wedge$: p: I got 80% marks
q: I got A Grade } $p \wedge q$ = I got 80% marks AND I got A grade.

$\vee$: p: } $p \vee q$ = I got 80% marks OR A grade.

p	$\neg p$
0	1
1	0

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

3. IMPLICATION

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

p: you get 60 marks in Gate

q: you will goto iit's.

$p \rightarrow q$: If you get 60 marks in Gate then you will goto iit's.

p: Today is monday (T/F)

q: $5 \times 2 = 10$ (T)

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

$p \rightarrow q$ is always a

The various ways of Representing $p \rightarrow q$ are:

- 1) If p then q . $\rightarrow$ Necessary condition for ' p ' is q .
- 2) ' p ' is sufficient for q . $\rightarrow q$ unless $\sim p$.
- 3) q if p . $\rightarrow$ ' p ' implies q !
- 4) q when p . $\rightarrow$ ' p ' only if q .
- $\rightarrow$ A sufficient condition for ' q ' is ' p '.
- $\rightarrow$ ' q ' whenever ' p '.
- $\rightarrow$ ' q ' is necessary for ' p '.
- $\rightarrow$ ' q ' follows from ' p '.

4. QUESTION ON IMPLICATION

- $p \rightarrow q$: If today is Friday then $2 \times 3 = 6$.
- Converse: $q \rightarrow p$: If $2 \times 3 = 6$ then today is Friday
- Contrapositive: $\sim q \rightarrow \sim p$: If $2 \times 3 \neq 6$ then today is not Friday
- Inverse: $\sim p \rightarrow \sim q$: If today is not Friday then $2 \times 3 \neq 6$.

i) $p \rightarrow q \neq q \rightarrow p$

$p \rightarrow q \equiv [\sim p \vee q]$

$\equiv (\sim q \rightarrow \sim p)$

$p \rightarrow q \equiv \sim q \rightarrow \sim p$ (conditional & contrapositive)

$q \rightarrow p \equiv \sim p \rightarrow \sim q$ (Inverse and Converse)

5. BI-CONDITIONAL

p	q	$p \leftrightarrow q$	$(p \rightarrow q) \wedge (q \rightarrow p)$
T	T	T	T
T	F	F	F
F	T	F	F
F	F	T	T

$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$

Various Representations of Biconditional are: if and only if

iff
it is necessary and sufficient.

precedence of operations : 1) Negation

2) Conjunction

$\Rightarrow \sim p \wedge q \vee i$

3) Disjunction

$\Rightarrow (\sim p) \wedge q) \vee i$

4) Implication

5) Bi Implication

6. Example 1

You can access the Internet only if you are a computer science major or you are not a freshman.

(26)

There are 3 statements here

p : you can access the Internet $\rightarrow p$

q : you are a computer science major

r : you are not a freshman.

$(q \vee r)$

$$\boxed{p \rightarrow (q \vee r)}$$

$\Rightarrow$ what ever before "only if" is called hypothesis and what is given after that is called conclusion.

1. Example 2

You cannot ride the roller coaster if you are under 4 feet tall unless you are older than 16 years.

p : you can ride the roller coaster

q : you are under 4 feet tall

r : you are older than 16 years.

Now, we can replace "unless" with "And" and Negate the statement "r".

$$(q \wedge \sim r)$$

$$(\sim p)$$

Before if
= conclusion

$$(q \wedge \sim r)$$

$\rightarrow$ present after if $\rightarrow$ Hypothesis.

$$p \rightarrow q \text{ (if } p \text{)}$$

conclusion Hypothesis
(After if)

$\therefore$ Final result = Hypothesis $\rightarrow$ Conclusion

$$\Rightarrow \boxed{(q \wedge \sim r) \rightarrow p}$$

8. EXAMPLE - 3

The automated reply cannot be sent (when) the file system is full

$\sim p$ when q

p : The automated reply can be sent

q : The file system is full

if $p \rightarrow q = \text{Q when P}$
 conclusion hypothesis

$$\therefore \boxed{q \rightarrow \sim p}$$

9. CONSISTENT SYSTEM

Determine whether these specifications are consistent.

- The diagnostic message is stored in the buffer or it is retransmitted
- The diagnostic message is not stored in the buffer
- "If the diagnostic message is stored in the buffer, then it is retransmitted."

1> The diagnostic message is stored in the buffer or it is retransmitted
 $P \quad Q = p \vee q$

2> The diagnostic message is not stored in buffer ($\sim p$)

3> If the diagnostic message is stored in the buffer then it is retransmitted. ($p \rightarrow q$)

$$1> (p \vee q)$$

$$2> (\sim p)$$

$$3> (p \rightarrow q)$$

Now we should find some assignment of truth values to 'p' and 'q' such that all the '3' will be true simultaneously

$p \vee q$

$\sim p$

$p \rightarrow q$

This should be true $\Rightarrow \sim p = T$

$$\Rightarrow \boxed{p = F}$$

Now, $p \vee q$ should be true $\Rightarrow \boxed{p = F}$ already known
 $\Rightarrow \boxed{q = T}$

SETS

1. SETS AND

Set: well-defined

Ex: $A = \{ \dots \}$

$S = \{ \dots \}$

Null set: Set

subset: If

of B.

$A = \{ \dots \}$

Note: For

Proper subset

proper subset

Note

2. POWER

Denoted by

$\Rightarrow$ If 'A' is

power set

Ex: $A = \{ \dots \}$

Sub

$\Rightarrow$ If a set

10. EQUIVALENCES

$$1 > p \rightarrow q = \sim p \vee q$$

$$2 > p \rightarrow q = \sim q \rightarrow \sim p$$

$$3 > p \vee q = \sim p \rightarrow q$$

$$4 > p \wedge q = \sim (q \rightarrow \sim p)$$

$$5 > \sim (p \rightarrow q) = p \wedge \sim q$$

$$6 > (p \rightarrow q) \wedge (p \rightarrow r) = p \rightarrow (q \wedge r)$$

$$7 > (p \rightarrow r) \wedge (q \rightarrow r) = (p \vee q) \rightarrow r$$

$$8 > (p \rightarrow q) \vee (p \rightarrow r) = p \rightarrow (q \vee r)$$

$$9 > (p \rightarrow r) \vee (q \rightarrow r) = (p \wedge q) \rightarrow r$$

$$10 > (p \leftrightarrow q) = (p \rightarrow q) \wedge (q \rightarrow p)$$

$$11 > \sim (p \leftrightarrow q) = (p \leftrightarrow \sim q)$$

11. DEMORGANS LAW

$$1 > \sim (p \vee q) = \sim p \wedge \sim q$$

$$2 > \sim (p \wedge q) = \sim p \vee \sim q$$

Ex: Ravi has a computer and a phone



If i apply Negation to this what will be the output??



let p: Ravi has a computer

q: Ravi has a phone

$$\text{Now, } \sim (p \wedge q) = \sim p \vee \sim q$$

⇒ Ravi doesnot have a ~~phone~~ computer
or Ravi doesnot have a phone

12. PROPOSITIONAL FUNCTION

→ proposition is a statement to which we can assign a truth value.

Ex: 2 is greater than 10 (false) — proposition.

{ x is greater than 10 } — cannot take T/F unless
variable predicate the value of x is known.

↓
This is called propositional
Function, $p(x)$

∴ $p(x)$: x is greater than 10

$p(2)$: 2 is > 10 (F)

$p(12)$: 12 is > 10 (T)

→ The propositional
function transforms itself
into proposition.

$p(x, y)$ computer x is connected to network y .

13. QUANTIFIERS

it $p(x): x > 2$

Domain: $\{1, 2, 3, 4\}$ $p(1)=F, p(2)=F, p(3)=T, p(4)=T$

Now, $\forall x p(x)$ is true, if $p(x)$ is true for all values of x .

6. Universal Quantifier/
universal quantification.

$\forall x p(x) = \text{false}$ in this case.

ex Now, let $p(x) = x > 2$

a) Domain = $\{1, 2, 3, 4\}$

$p(1)=F, p(2)=F, p(3)=T, p(4)=T \therefore \exists x p(x)$ is true if there exists atleast one value of x in the domain for which $p(x) = \text{True}$

Opt In this case $\exists x p(x) = \text{True}$

Existential Quantifier.

14. RELATION BETWEEN THE TWO QUANTIFIERS

Let if $\forall x p(x)$ is true then $\exists x p(x)$ is True

con if $\forall x p(x)$ is false then $\exists x p(x)$ is (false/True)

a) if $\exists x p(x)$ is true then $\forall x p(x)$ is true

opt if $\exists x p(x)$ is false then $\forall x p(x)$ is false

$\therefore$ The precedence of $\forall, \exists$ will be higher than any other logical connectives.

$$\forall x p(x) \vee Q(x) \Rightarrow [\forall x p(x)] \vee Q(x).$$

15. DISTRIBUTING QUANTIFIERS

$$1) \forall x (x+y+z) \cong (\forall x (x)) + y+z = 10$$

$$2) \exists x p(x) \vee \forall x Q(x) \cong [\exists x p(x)] \vee [\forall x Q(x)]$$

$$3) \forall x (p(x) \wedge Q(x))$$

$$\text{Domain} = \{x_1, x_2\} = p(x_1) \wedge Q(x_1) \wedge Q(x_2) \wedge Q(x_2) \therefore \forall \text{ can be}$$

SETS

SETS AND

Set: well defined

Ex: $A = \{1, 2\}$

$S = \{s\}$

Null set: Set

subset: If

of B.

$A = \{f\}$

Note: For

Proper subset

proper subset

Note:

2. POWER SET

Denoted by

$\Rightarrow$ If A is

power set

Ex: $A = \{a\}$

subset

$\Rightarrow$ If a set

④ $\forall x$ cannot be "distributed over disjunction." (35)

$$\forall x (p(x) \vee q(x)) = [p(x_1) \vee q(x_1)] \wedge [p(x_2) \vee q(x_2)]$$

$$D: \{x_1, x_2\}$$

$$[\forall x p(x)] \vee [\forall x q(x)] = [p(x_1) \wedge p(x_2)] \vee [q(x_1) \wedge q(x_2)]$$

Not equal

⑤ $\exists x (p(x) \vee q(x)) = [\exists x p(x)] \vee [\exists x q(x)] \rightarrow$ There exists can be distributed over disjunction.

16. QUANTIFIERS WITH NEGATION

→ Every student in the class has taken a course in CS

$p(x)$: x has taken a course in CS

$\forall x p(x)$:

x = students of the class

Now, $\sim(\text{statement 1})$ = There exist atleast one student in the class who has not taken the course in CS.

Let

conv

a)

opv

$x_1 =$

17. EXAMPLES ON NEGATING THE QUANTIFIERS

→ There is an honest politician

$H(x)$: x is honest

$\exists x H(x)$: There is an honest politician

x : {All politicians}

Now, $\sim(\exists x H(x)) = \forall x \sim(H(x))$

= All the politicians are dishonest.

→ All Americans eat cheese burgers.

$C(x)$: x eats cheese burgers $\Rightarrow \forall x C(x)$ and x : {All Americans}, Now

$\sim(\forall x C(x)) = \exists x \sim(C(x)) \Rightarrow$ "There exists an American who don't eat cheese burgers."

SETS

1. SETS AND

Set: well-def

Ex: $A = \{1, 2, 3\}$

$S = \{set\}$

Null set: Set with

subset: If every

of B.

$A = \{1, 2, 3\}$

Note: For every

Proper subset: Any

proper subset of

Note: If A

POWER SET

denoted by $P(A)$

If A is finite set

power set of A . If

$A = \{a, b\}$

subsets of $A =$

$P(A) = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$

a set contains

18. IMPORTANT EXAMPLES

$$1) \sim \forall x (p(x) \wedge q(x))$$

$$\Rightarrow \sim \forall x [R(x)] \text{ where } R(x) = p(x) \wedge q(x)$$

$$\Rightarrow \exists x \sim (R(x))$$

$$6. \Rightarrow \exists x \sim (p(x) \wedge q(x))$$

wh

ex

$$\Rightarrow \exists x [\sim p(x) \vee \sim q(x)]$$

$$a) \Rightarrow \sim \forall x (p(x) \rightarrow q(x))$$

$$\Rightarrow \sim \forall x (\sim p(x) \vee q(x))$$

F(1)

$$\Rightarrow \exists x [p(x) \wedge \sim q(x)]$$

NOTATION WITH NEGATION

Statement 4

$$\text{opt: } 3) \sim \exists x (p(x) \wedge q(x)) = \forall x (\sim p(x) \vee \sim q(x)) \equiv \forall x (p(x) \rightarrow \sim q(x))$$

$$4) \forall x (p(x) \rightarrow \sim q(x)) = \forall x (\sim p(x) \vee \sim q(x))$$

7. €

Let

cons:

a) x

vm

=

opr

19. TRANSLATING ENGLISH STATEMENTS TO PROPOSITIONAL FUNCTION

Every student in the class has taken the course operating system.

$O(x)$: 'x' has taken the course OS.

x : {students in the class.}

$\forall x O(x)$

Now let $S(x)$: x is a student in the class.

$\therefore$ If 'x' is a student in the class then he has taken OS

$$\forall x (S(x) \rightarrow O(x)) \quad (\text{domain is expanded here})$$

For all people in the world

If he has a student in the class then he has taken the course OS.

x : {All the people in the world}

Hint: Now, if you see the word Every in the question the option must contain $\forall$ and $\rightarrow$.

$$\text{Now, } \sim \forall x (S(x) \rightarrow O(x)) = \exists x (S(x) \wedge \sim O(x)) \rightarrow \text{check}$$

SETS

SETS AND SUBSETS

Set: well-defined

Ex: $A = \{1, 2, 3, 4\}$

S = {set of}

Null set: Set with

subset: If every

of B.

$A = \{1, 2, 3\}$

Note: For every

proper subset: Any

proper subset of

Note: If A

POWER SET

denoted by $P(A)$

If A is finite set

power set of A:

x : $A = \{a, b\}$

subsets of A =

$P(A) = \{\emptyset, A\}$

If a set contains

Now, if the questions are given of this form $T(n) = 3T(n/2)$
it as $T(n)$

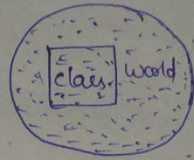
20. TRANSLATION CONTINUED

Some student in the class has taken OS course

$O(x)$: x has taken OS.

$\Rightarrow \exists x O(x)$

x : students in the class



Now, $S(x)$: x is student in the class

$\exists x (S(x) \wedge O(x)) \Rightarrow$ Now, if you see "some" then Ans should contain ($\exists$ and AND).

Now, $\sim [\exists x (S(x) \wedge O(x))]$

$\sim$ (some student in the class has taken OS course) = (No student in the class has taken OS course)

$$\therefore \sim [\exists x (S(x) \wedge O(x))]$$

$$= \forall x [\sim S(x) \vee \sim O(x)]$$

$$= \forall x [S(x) \rightarrow \sim O(x)]$$